Lecture 2: Hitting Families, Random Walks

Tuesday, August 6, 2019 9:31 AM

Follow along on
http://tiny.cc/91ssaz
https://1drv.ms/o/s!Aufrpyzk MX1pcZcVPUGorRH7xMI

A banquet is fair if for every pair i,j of ninjas there is a course in which i is fed before j and another in which j is fed before i

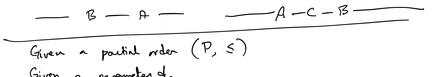
What about 3-fair banquets?
For every triple (i,j,k) there is a course in which i is fed before j and j before k.

Theorem: For every d, there is a d-fair banquet with $d!\, d \log n$ rounds.

Pf: Fix a d-tuple of ninjas.
Pr that a random perm "hits" the d-tuple is $\frac{1}{d!}$
Pr that ... does not hit, $1 - \frac{1}{d!}$
Pr that K ... $\left(1 - \frac{1}{d!}\right)^K$

The prob that K random perm. do not form a fair banquet is $\leq \binom{n}{d} \left(1 - \frac{1}{d!}\right)^K$

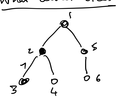
If $K > d!\, d \ln n$ this ≤ 1



Given a partial order $(P, \leq)$
Given a parameter d.
Coverage goal: For every d-tuple of elements $p_1, \ldots, p_d$ from P there is a test that orders $p_1 < p_2 < \cdots < p_d$
If this is allowed by the partial order.

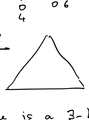
What about trees

$d = 2$ We need two orderings

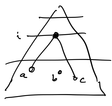


1 2 3 4 5 6
1 5 6 2 4 3

$d = 3$



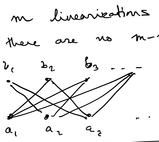
There is a 3-hitting family with $2h$ schedules.
for each $i \in \{0, \ldots, h-1\}$
ldfs blocking right at i; ldfs of the rest
ldfs blocking left at i; ldfs of the rest
rdfs blocking left at i; rdfs the rest
rdfs blocking right at i; rdfs the rest



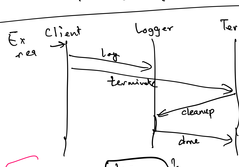
Theorem: For any $d \geq 3$ there is a family with $\exp[d]\, d!\, h^{d-1}$ schedules.

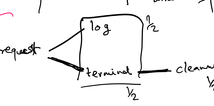
Order Dimension of a partial order

Given a p.o. $(P, \leq)$, the order dimension is m if there are m linearizations of P whose intersection is P. (and there are no $m-1$ lin. with this property)

Ex: 

Ex: Show this has order dim n.

Ex 



$\frac{1}{2}$ request log
$\frac{1}{2}$ Terminate cleanup done

Online Dimension Problem

Program: Reveals a p.o. one element at a time, in "upgrowing" manner

Scheduler: Maintain a chain partitioning of the p.o.

C_1
C_2
$\vdots$
C_k

A chain is a linearly ordered subset.
$a_1 a_2 \cdots a_{i_k}$ s.t. $a_1 \leq a_2 \leq a_3 \cdots \leq a_k$

Suppose that the underlying partial order has width w.

Dilworth's Theorem: Any p.o. of width w has a chain partition with w chains.

Theorem: In the online chain partitioning game, scheduler can always find a chain partitioning with $\binom{w+1}{2}$ chains.

A randomized algorithm to sample from a 2-hitting family:
(1) Maintain an online chain partitioning
(2) Assign a random priority to each chain
(3) Always pick the highest priority chain that has an enabled event.

Theorem: This algorithm samples a 2-hitting family with prob. at least $\frac{1}{w^2} \frac{1}{n^{d-2}}$.

Fuzzing

http://www.fuzzing-book.org

OSS-Fuzz

T = Initial random set of inputs
repeat
  t = pick(T)
  t' = mutate the input t
  run the program on t'
  if the program crashes return Bug
  if t' is "interesting" add t' to T.
until enough bugs
or
time to go home